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(Pages : 3) **6418**

Reg. No. : ....

Name : ....

Fifth Semester B.Tech. Degree Examination, November 2014
(2008 Scheme)
08.501 : ENGINEERING MATHEMATICS – IV
Complex Analysis and Linear Algebra (T A)

Time : 3 Hours Max. Marks : 100

***Instructions** : Answer **all** questions from Part **A** and **one** full question from **each** Module of Part **B**.*

PART – A

(10×4= 40 Marks)

1. Show that f(z) = log z is differentiable except at z = 0 and find its derivative.
2. Show that the function $u = x^3 - 3xy^2 + y$ can be the real part of an analytic function and find the corresponding imaginary part.
3. Show that an analytic function with a constant argument is a constant.
4. Find the image of the interior of the triangle bounded by x = 1, y = 0 and y = x under $w = z^2$.
5. Evaluate $\int_C \frac{z+2}{z} dz$ where C is $|z| = 2$ with $-\pi \le \theta \le \pi$.
6. Expand $\frac{e^2 z}{(z-1)^3}$ about z = 1.
7. Evaluate $\int_{|z|=2} \frac{dz}{z^3(z+4)}$.

P.T.O.

8. Difine the subspace of a vector space. Show that

$H = \{(a + 3b, a - b, 2a - b, 4b)^T = a, b \in R\}$ is a subspace of R^4.

9. Define a Basis. Find a basis of the subspace consisting of the solutions of the homogeneous system $x_1 + 2x_2 = 0, 2x_1 - x_2 + 3x_3 = 0$. What is its dimension ?

10. Find a least square solution of the in consistent system

$$\begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

PART – B

(3×20= 60 Marks)

Module – I

11. a) Show that $f(z) = \frac{xy^2}{x^2 + y^2}$, $z \neq 0$ and $f(0) = 0$ satisfy CR equations, but not differentiable at $z = 0$.

b) If $\psi = x^2 - y^2 + \frac{x}{x^2 + y^2}$, find the potential function ϕ.

c) Show that the transformation $w = z + \frac{1}{z}$ maps the circle $|z| = C$ into an ellipse. Discuss the case when $C = 1$.

12. a) If f(z) is analytic show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4 |f'(z)|^2$.

b) Find the analytic function $f(z) = u + iv$ if $u - v = (x - y)(x^2 + 4xy + y^2)$.

c) Find the bilinear transformation which maps $(-i, 0, i)$ into $(-1, i, 1)$.

Module – II

13. a) Evaluate $\int_C \frac{\cos \pi z^2}{(z-1)(z-2)} dz$ where C is $|z| = 3$.

b) If $f(a) = \int_C \frac{3z^2 + 7z + 1}{z-a} dz$ where C is the circle $x^2 + y^2 = 4$, find $f(z)$, $f'(1-i)$ and $f''(1-i)$.

c) Find the Laurent's series expansion of $\frac{1}{z-z^3}$ in $1 < |z+1| < 2$.

14. a) Find the poles and residues of $f(z) = \tan z$.

b) Evaluate $\int_0^{\pi} \frac{d\theta}{a + \cos\theta}$ $(a > 1)$ by contour integration.

c) Prove that $\int_0^{\infty} \frac{x^2}{(x^2+1)^2} dx = \frac{\pi}{4}$.

Module – III

15. a) Solve the following equation AX = B by using LU factorization of A.

$x + y + 2z = 3$

$3x + 4y + 8z = 10$

$-2x + 2y + z = 7.$

b) Define the null space and column space of an m × n matrix A. Find a basis for the row space, column space and null space of A. Find also their dimensions.

c) Find an orthonormal basis for the subspace spanned by (1, 0, 1, 2), (2, 1, 0, 2) and (1, – 1, 0, 1) in $\mathbb{R}^4$

16. a) Find maxima or minima of

$f(x) = x_1^2 + x_2^2 - 4x_1 - 2x_2 + 6$.

b) Find a singular value decomposition of the matrix $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$.

c) Find the projection matrix of $u = \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}$ and hence find the projection of $v = (1, 1, 1)$ on u.
